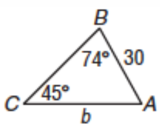
The Law of Sines & Cosines Notes
Honors Geometry

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **In $\triangle ABC$, find b.**



$\frac{\sin C}{c} = \frac{\sin B}{b}$ Law of Sines

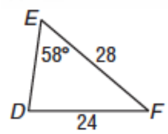
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$ $m\angle C = 45$, $c = 30$, $m\angle B = 74$

$b \sin 45° = 30 \sin 74°$ Cross multiply.

$b = \frac{30 \sin 74°}{\sin 45°}$ Divide each side by sin 45°.

$b \approx 40.8$ Use a calculator.

Example 2 **In $\triangle DEF$, find $m\angle D$.**



$\frac{\sin D}{d} = \frac{\sin E}{e}$ Law of Sines

$\frac{\sin D}{28} = \frac{\sin 58°}{24}$ $d = 28$, $m\angle E = 58$, $e = 24$

$24 \sin D = 28 \sin 58°$ Cross multiply.

$\sin D = \frac{28 \sin 58°}{24}$ Divide each side by 24.

$D = \sin^{-1} \frac{28 \sin 58°}{24}$ Use the inverse sine.

$D \approx 81.6°$ Use a calculator.

Exercises

Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find a.

2. If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.

3. If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$.

4. If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find b.

5. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find a.

6. If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$.

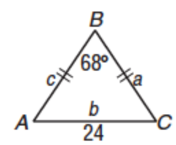
Use the Law of Sines to Solve Problems

You can use the **Law of Sines** to solve some problems that involve triangles.

Law of Sines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Example

Isosceles $\triangle ABC$ has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.



The vertex angle is 68°, so the sum of the measures of the base angles is 112 and $m\angle A = m\angle C = 56$.

$$\frac{\sin B}{b} = \frac{\sin A}{a} \quad \text{Law of Sines}$$

$$\frac{\sin 68°}{24} = \frac{\sin 56°}{a} \quad m\angle B = 68,\ b = 24,\ m\angle A = 56$$

$$a \sin 68° = 24 \sin 56° \quad \text{Cross multiply.}$$

$$a = \frac{24 \sin 56°}{\sin 68°} \quad \text{Divide each side by } \sin 68°.$$

$$\approx 21.5 \quad \text{Use a calculator.}$$

The triangle is isosceles, so $c = 21.5$.

The perimeter is 24 + 21.5 + 21.5 or about 67 centimeters.

Exercises

Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.

2. Two radar stations A and B are 32 miles apart. They locate an airplane X at the same time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures 52°. How far is the airplane from each station?

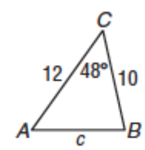
3. A civil engineer wants to determine the distances from points A and B to an inaccessible point C in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points A and B are 82.0 feet apart, find the distance from C to each point.

4. A ranger tower at point A is 42 kilometers north of a ranger tower at point B. A fire at point C is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°, which ranger tower is closer to the fire? How much closer?

The Law of Cosines Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

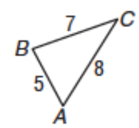
Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $\quad b^2 = a^2 + c^2 - 2ac \cos B$ $\quad c^2 = a^2 + b^2 - 2ab \cos C$

Example 1 **In $\triangle ABC$, find c.**



$c^2 = a^2 + b^2 - 2ab \cos C$	Law of Cosines
$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48°$	$a = 12$, $b = 10$, $m\angle C = 48$
$c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48°}$	Take the square root of each side.
$c \approx 9.1$	Use a calculator.

Example 2 **In $\triangle ABC$, find $m\angle A$.**



$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$	$a = 7$, $b = 5$, $c = 8$
$49 = 25 + 64 - 80 \cos A$	Multiply.
$-40 = -80 \cos A$	Subtract 89 from each side.
$\frac{1}{2} = \cos A$	Divide each side by -80.
$\cos^{-1}\frac{1}{2} = A$	Use the inverse cosine.
$60° = A$	Use a calculator.

Exercises

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $b = 14$, $c = 12$, and $m\angle A = 62$, find a.

2. If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$.

3. If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$.

4. If $a = 20$, $c = 25$, and $m\angle B = 82$, find b.

5. If $b = 18$, $c = 28$, and $m\angle A = 59$, find a.

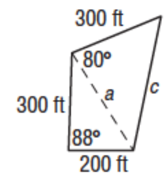
6. If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$.

Use the Law of Cosines to Solve Problems

You can use the **Law of Cosines** to solve some problems involving triangles.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $\quad b^2 = a^2 + c^2 - 2ac \cos B$ $\quad c^2 = a^2 + b^2 - 2ab \cos C$

Example **Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.**



Use the Law of Cosines to find the value of a.

$a^2 = b^2 + c^2 - 2bc \cos A$ — Law of Cosines

$a^2 = 300^2 + 200^2 - 2(300)(200) \cos 88°$ — $b = 300$, $c = 200$, $m\angle A = 88$

$a = \sqrt{130{,}000 - 120{,}000 \cos 88°}$ — Take the square root of each side.

≈ 354.7 — Use a calculator.

Use the Law of Cosines again to find the value of c.

$c^2 = a^2 + b^2 - 2ab \cos C$ — Law of Cosines

$c^2 = 354.7^2 + 300^2 - 2(354.7)(300) \cos 80°$ — $a = 354.7$, $b = 300$, $m\angle C = 80$

$c = \sqrt{215{,}812.09 - 212{,}820 \cos 80°}$ — Take the square root of each side.

≈ 422.9 — Use a calculator.

The perimeter of the land is $300 + 200 + 422.9 + 200$ or about 1223 feet.

Exercises

Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.

2. A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.

3. An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0°, and 40 miles from the other location with an angle of elevation of 21.8°. How far apart are the two locations?

4. A ranger tower at point A is directly north of a ranger tower at point B. A fire at point C is observed from both towers. The distance from the fire to tower A is 60 miles, and the distance from the fire to tower B is 50 miles. If $m\angle ACB = 62$, find the distance between the towers.